Sensitivity Analysis of Automatic Flight Control Systems Using Singular-Value Concepts

Alfredo Herrera-Vaillard*
Institute for Electrical Research, Cuernavaca, Mexico
and
James Paduano† and David Downing‡
University of Kansas, Lawrence, Kansas

Presented is a sensitivity analysis that can be used to judge the impact of vehicle dynamic model variations on the relative stability of multivariable continuous closed-loop control systems. The sensitivity analysis uses and extends the singular-value concept by developing expressions for the gradients of the singular value with respect to variations in the vehicle dynamic model and the controller design. Combined with a priori estimates of the accuracy of the model, the gradients are used to identify the elements in the vehicle dynamic model and controller that could severely impact the system's relative stability. The technique is demonstrated for a yaw/roll damper system designed for a business jet.

Nomenclature

A	= system matrix
$a(i,j)$	= matrix element
B	= system matrix
$\bar{C}$	= feedback matrix
GM	= gain margin, dB
I	= identity matrix
j	$=\sqrt{-1}$
K	= matrix of feedback gains
$K_{\delta A}, K_{\delta R}$	= feedback gains for aileron and rudder
L_β	= rolling moment due to sideslip
L_P	= rolling moment due to roll rate
$L_{\delta A}$	= rolling moment due to aileron deflection
N_β	= yawing moment due to sideslip
$N_{\delta R}$	= yawing moment due to rudder deflection
p	= airplane roll rate, s^{-1}
p	= a system or controller parameter
PM	= phase margin, deg
r	= airplane yaw rate, s^{-1}
Re $(\cdot)$	= real part of complex matrix $(\cdot)$
s	= Laplace transform variable
T	= output matrix
tr$(\cdot)$	= trace of matrix $(\cdot)$
U	= control input vector
U_1	= steady-state airplane speed
u_i, v_i	= left and right normalized singular vectors of the return difference matrix corresponding to the singular value σ_i
Y_r	= dimensional stability derivative relating side force due to yaw rate, ft/s
X	= state variable vector
Z	= output variable vector
$\underline{\sigma}(\cdot)$	= minimum singular value of $(\cdot)$
ϕ	$=(Is-A)^{-1}$
δ_A, δ_R	= aileron and rudder deflections
β	= sideslip angle in rads
ϕ	= roll angle in rads
ζ	= washout filter state
$(\cdot)^*$	= conjugate transpose of matrix $(\cdot)^*$

Subscript

c	= commanded

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*Research Engineer (formerly Graduate Student, University of Kansas). Member AIAA.

†Graduate Research Assistant, Flight Research Laboratory. Student Member AIAA.

‡Professor of Aerospace Engineering, Flight Research Laboratory. Member AIAA.

Introduction

THE performance of a flight control system design is strongly affected by the accuracy of the model of the open-loop vehicle used during the design process. To assure acceptable performance of the design on the real vehicle, the designer can either attempt to build insensitivity to model errors into the design (robust control) or require additional wind tunnel or flight testing to assure the model's fidelity. This paper presents a procedure the designer can use to determine whether or not the model is adequate or what additional testing is required.

The sensitivity analysis assumes that a continuous feedback controller has been designed based upon a preliminary linear model of the vehicle. The technique used to design the controller is not restricted and can use either classical or optimal design procedures. The sensitivity analysis then provides information on which parameter variations in the vehicle model or the feedback controller design have the largest potential impact on the closed-loop system's relative stability.

The sensitivity analysis bases its measure of relative stability on the singular-value analysis. Much work has recently been done on the application of singular-value analysis to various control design problems. Singular values have been used to find the upper bounds on the norm of the plant model perturbation that will not destabilize the closed-loop system.[1,2] Also, singular-value gradients with respect to controller parameters combined with optimization techniques have been used to increase the robustness of the closed-loop design.[3] The singular-value analysis is particularly useful to the control designer in that it presents the relative stability of multiloop control systems in the familiar terms of gain margin and phase margin.[1,4]

This work has shown singular values to be a useful robustness measure with many desirable properties, in spite of its inherently conservative prediction of classical gain and phase margin.[5] Singular-value analysis is gaining wide acceptance and accessibility and, as such, presents a workable tool for first-order sensitivity studies.

The present paper extends the singular-value gradient work[3] by deriving singular-value gradients with respect to the open-loop vehicle model as well as the controller design. These gradients are then applied to the identification of key model parameters. The method is illustrated for a business jet yaw/roll damper system designed using classical techniques.

Analysis

Problem Formulation

Given the time domain representation of the multivariable system as

$$\dot{X} = AX + BU \tag{1}$$

$$Z = TX \tag{2}$$

where X is the state vector, U the control vector, and Z the output vector.

Also, it is assumed that there is a feedback control design given by

$$U = -\bar{C}X + R, \quad \bar{C} = KT \tag{3}$$

where K is the matrix of controller feedback gains and R is the command vector.

Taking the Laplace transform of Eqs. (1-3) and rearranging, U can be written

$$U(s) = [I + \bar{C}(Is - A)^{-1}B]^{-1}R(s) \tag{4}$$

The matrix $[I + \bar{C}(Is - A)^{-1}B]$ is called the return difference matrix for the loop broken at the plant input. Note that this is the inverse of the matrix that relates the output to the input. Thus, it can be seen from Eq. (4) that, using as the definition of stability that a bounded input produces a bounded control, the closed-loop system will be unstable if the return difference matrix is singular. Because the minimum singular value of a matrix is a reliable measure of that matrix's nearness to singularity,[6] the minimum singular value of the return difference matrix represents a measure of the relative stability of the closed-loop system. That is, nearness to the stability boundary can be quantified if the closed-loop system is known to be stable. The relationships between the minimum singular value and the multiloop gain and phase margin are given by[7]

$$\mathrm{GM} = \left[\frac{1}{1 + \underline{\sigma}[I + \bar{C}(Is - A)^{-1}B]}\right]_{s = jw} \tag{5}$$

$$\mathrm{PM} = \pm\cos^{-1}\left\{1 - \frac{\underline{\sigma}^2[I + \bar{C}(Is - A)^{-1}B]}{2}\right\}_{s = jw} \quad \text{for } \underline{\sigma} \le 1 \tag{6}$$

Similar equations, which improve the conservatism of the predictions in Eqs. (5) and (6), are available,[5] but are not useful when studying gradients (which will be described in the next section) because they are based on the inverse return difference matrix. What is important for the purposes of the present paper is that an easily computed, reliable measure of relative stability is available. Some characteristics of the singular values are summarized in Table 1.

Derivation of Singular-Value Gradients

To determine the effect that changes in the system and controller parameters have on the relative stability of a closed-loop system, the derivatives of the singular values with respect to elements of the A, B, and $\bar{C}$ matrices must be evaluated. It can be shown[3,4] that for a distinct singular value σ_i of a complex return difference matrix, the gradient with respect to a real parameter p is given by the scalar relation

$$\frac{\partial \sigma_i}{\partial p} = \mathrm{Re}\left[u_i^* \frac{\partial(I + \bar{C}\phi B)}{\partial p} v_i\right] \tag{7}$$

where $\phi = (Is - A)^{-1}$ and v_i and u_i are, respectively, right and left normalized singular vectors of the return difference matrix corresponding to the singular value σ_i.

Introducing the trace operation, Eq. (7) can be written as

$$\frac{\partial \sigma_i}{\partial p} = \mathrm{Re}\cdot\mathrm{tr}\left[\frac{\partial(I + \bar{C}\phi B)}{\partial p} v_i u_i^*\right] \tag{8}$$

Equation (8) can then be expanded in terms of the fundamental matrices A, B, and $\bar{C}$ as

$$\frac{\partial \sigma_i}{\partial p} = \mathrm{Re}\cdot\mathrm{tr}\left[\left\{\frac{\partial \bar{C}}{\partial p}\phi B + \bar{C}\phi\frac{\partial B}{\partial p} + \bar{C}\phi\frac{\partial A}{\partial p}\phi B\right\} v_i u_i^*\right] \tag{9}$$

It is now possible to obtain expressions for the gradients of all the elements of A, B and $\bar{C}$.[8] The results are

$$\frac{\partial \sigma_i}{\partial A^T} = \mathrm{Re}\left[\phi B v_i u_i^* \bar{C}\phi\right] \tag{10}$$

Table 1 Characteristics of singular-value analysis

1) Singular-value analysis is based on the Euclidean norm of a matrix.
2) The singular values of A are defined as the positive square root of the eigenvalues of (A^*A).
3) Minimum singular-value plot alone will not identify if the system is stable or unstable.
4) Minimum singular value of A is a measure of how close A is to singularity.
5) The modes of a system are identified by local minimum of the sigma plot.

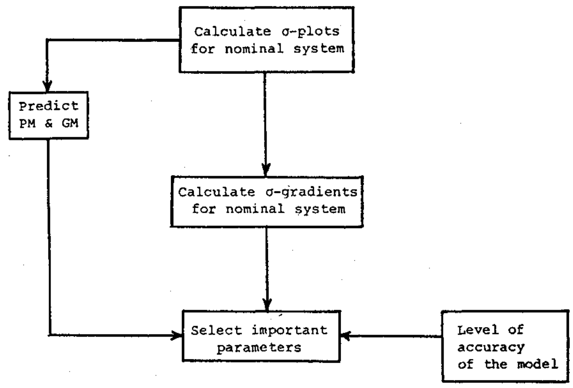

Fig. 1 Multiloop sensitivity analysis technique.

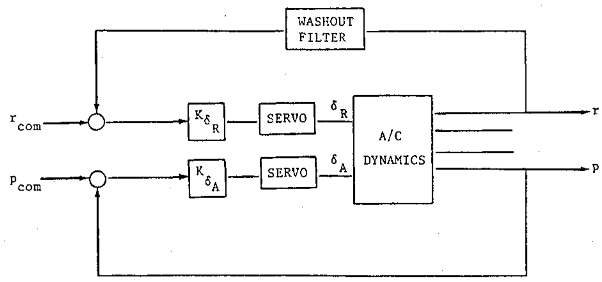

Fig. 2 Block diagram for yaw/roll damper system for a typical business jet.

$$\frac{\partial \sigma_i}{\partial B^T} = \mathrm{Re}\left[v_i u_i^* \bar{C}\phi\right] \tag{11}$$

and

$$\frac{\partial \sigma_i}{\partial \bar{C}^T} = \mathrm{Re}\left[\phi B v_i u_i^*\right] \tag{12}$$

where it is convenient to deal with the gradient with respect to the transpose of the A, B, and $\bar{C}$ matrices.

Equations (10–12) can be used to evaluate the singular-value gradients with respect to elements of the system (A and B) and controller ($\bar{C}$) matrices. Note that the gradients, like the singular values, are a function of frequency; and therefore, a singular-value gradients plot (or sigma gradients plot) can be obtained for a range of frequencies. It should be noted that the information necessary to obtain the gradients (u_i, v_i and ϕ) is already available in the calculation of the singular-values plot and, thus, little additional effort is needed to calculate the singular-value gradients.

Sensitivity Analysis Procedure

Combining the singular-value gradients developed in the previous section with the singular-value analysis and additional engineering data, it is possible to define a sensitivity analysis that will help the control designer judge if the vehicle model is adequate (i.e., an expected model variation will not have a significant impact on system performance) or what further model refinements are required. The steps in the procedure are illustrated in Fig. 1.

From the singular-value plot, the designer can identify the minimum singular value and judge the relative stability of the nominal design in terms of phase or gain margin. As a rule of thumb, a system whose minimum singular value is 0.5 will have at least a $-3.5 \leq \mathrm{GM} \leq 6$ dB and $\mathrm{PM} = \pm 29$ deg, while systems with minimum singular values greater than 1.0 are very stable systems with at least a $-6 \leq \mathrm{GM} \leq \infty$dB and a $\mathrm{PM} = \pm 60$ deg. Also, from the singular-value plots, the designer will be able to identify the critical frequencies or range of frequencies where the singular value is close to zero. Next, a singular-value gradient plot will be constructed for each element of the A, B, and $\bar{C}$ matrices. From those plots, the designer can identify the parameter(s) that have large gradients and at what frequencies the large gradients occur. The last piece of information required in the sensitivity analysis is an estimate of the accuracy of each element in the vehicle model and controller. This will be based on engineering judgment and the a priori estimate of the model's refinement.

Two forms of presentation based on the raw singular values and singular-value gradient information are suggested. First, if as is common in aircraft applications, the magnitudes of the elements of the A and B matrices can differ by several orders of magnitude, it is recommended that the singular-value gradients be scaled by the magnitude of each element. This would then produce singular-value gradients relative to a percentage change in each element. A second form of presentation that is useful is a linear prediction of the percent change in an element required to produce a fixed percentage change in the singular value: e.g., 10% reduction in the singular value. Once calculated, this can be quickly compared with the estimated model element accuracy to identify important elements.

Several words of caution should be noted. First, although the calculation of the singular-value gradients produces useful information, the control designer must have a thorough understanding of his system (vehicle dynamics and controller) to correctly interpret the results. This will be illustrated in the following example. Second, since these are first-order gradients and, in general, the singular values do not vary linearly with parameter variations, caution should be exercised when predictions require large changes in either the parameters or the singular values. Because the purpose of the sensitivity analysis is to give a list of candidate parameters for further study, nonlinearities do not conflict with the basic intent of the analysis. Finally, gradients with respect to other singular values of the system besides the minimum singular value may also be important. For instance, if a parameter variation causes a singular value that is not the minimum to become the minimum, then that parameter is potentially an important one, depending on the magnitude of the parameter variation required to do so.

Example

To demonstrate the sensitivity analysis, the design of a multiloop yaw/roll damper system of a typical business jet[9] is

Table 2 State-space representation of yaw/roll damper system for a typical business jet

$$\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \\ \dot{\delta}_R \\ \dot{\zeta} \\ \dot{\delta}_A \end{bmatrix} = \begin{bmatrix} -0.111 & 0 & -0.995 & 0.1594 & 0.0209 & 0 & 0 \\ -2.133 & -0.534 & 0.416 & 0 & 0.3207 & 0 & 2.441 \\ 1.0168 & -0.0515 & -0.1875 & 0 & -0.7051 & 0 & -0.3416 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -0.25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -10 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \\ \delta_R \\ \zeta \\ \delta_A \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 10 & 0 \\ 0 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} \delta_{R_C} \\ \delta_{A_C} \end{bmatrix}$$

$$\begin{bmatrix} \delta_{R_C} \\ \delta_{A_C} \end{bmatrix} = - \begin{bmatrix} 0 & 0 & -K_{\delta_R} & 0 & 0 & 0.25K_{\delta_R} & 0 \\ 0 & K_{\delta_A} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \\ \delta_R \\ \zeta \\ \delta_A \end{bmatrix} \qquad \begin{matrix} K_{\delta_R} = 1.0 \\ K_{\delta_A} = 0.2 \end{matrix}$$

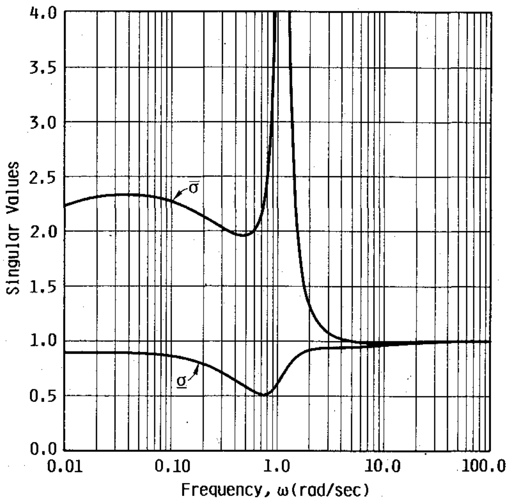

Fig. 3 Plot of minimum and maximum singular values for the nominal yaw/roll damper system.

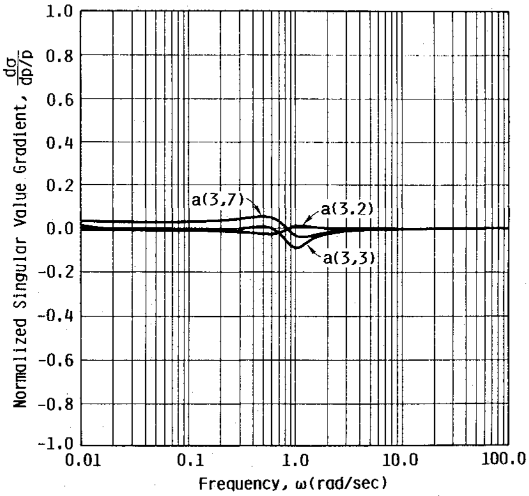

Fig. 5 Singular-value gradients with respect to elements $a(3,2)$, $a(3,3)$, and $a(3,7)$ of matrix A of the yaw/roll damper SAS.

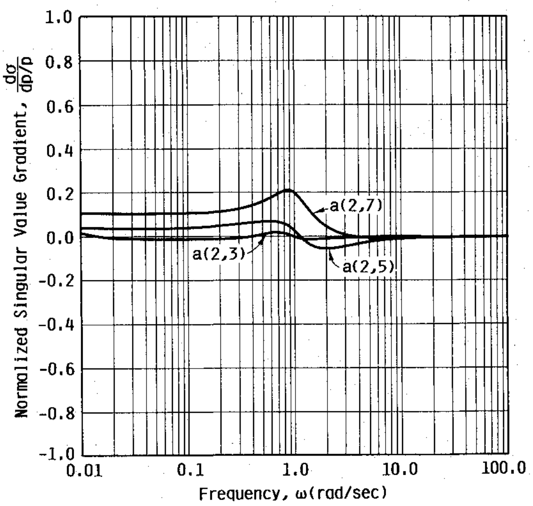

Fig. 4 Singular-value gradients with respect to elements $a(2,3)$, $a(2,5)$, and $a(2,7)$ of matrix A of the yaw/roll damper SAS.

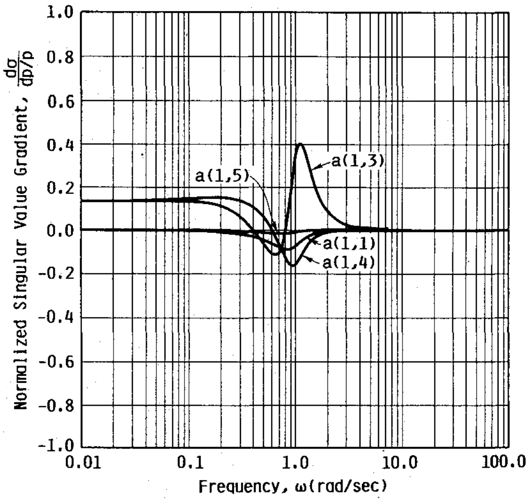

Fig. 6 Singular-value gradients with respect to elements $a(1,1)$, $a(1,3)$, $a(1,4)$, and $a(1,5)$ of matrix A of the yaw/roll damper SAS.

considered. See Fig. 2. The state-space representation of the system and control law is shown in Table 2, where servo dynamics and a washout filter have been included. The present analysis was limited to the consideration of variations only of elements in the A matrix. There are two singular values of the return difference matrix for this system; both are plotted in Fig. 3 for a yaw-damper loop gain $K_{\delta_R} = 1.0$, and roll-damper loop gain $K_{\delta_A} = 0.2$. No claims are made about the performance or robustness qualities of this design; it is used only as an example. The minimum singular value is $\underline{\sigma} = 0.502$, which using Eqs. (5) and (6), yields a $-3.24 \leq \text{GM} \leq 5.22$ dB and PM $= \pm 29.1$ deg. The minimum singular value occurs at 0.76 rad/s. It can be seen that for frequencies less than 0.1 rad/s and more than 3.0 rad/s, the minimum singular value is close to 1.0. The frequency range of primary interest for the singular-value gradients analysis would therefore be $0.1 \leq \omega \leq 3$rad/s.

Prior to the generation of singular-value gradients of A, the designer's knowledge of the application should be exercised. On examinaton of the A matrix in Table 2, it is seen that the fifth and seventh rows contain the servo dynamics and the sixth row contains the washout filter dynamics. For this example, it is assumed that these are known perfectly and will not vary. Therefore, it is not necessary to calculate their singular-value gradients. If, in the problem of interest, filters or servos were not accurately known, their variability could easily be included in the analysis. Also, it is important to note that the fourth row is a kinematic equation that relates $\dot{\phi}$ to aircraft roll rate p. This also cannot vary, so there is no need to calculate the corresponding gradients. Therefore, the problem has been reduced to finding the singular-value gradients for the elements in only the first three rows, which correspond to the stability and control derivatives of the aircraft. The singular-value gradient plots are given in Figs. 4–7. Plots are not included for elements $a(1,1)$, $a(1,6)$, $a(1,7)$, $a(2,4)$, $a(2,5)$, $a(3,4)$, and $a(3,5)$ in that their gradients were virtually zero for all frequencies. Table 3 summarizes the results of the singular value and singular-value gradient plots. Column 1 lists the 14 elements evaluated, column 2 indicates the frequency at which the maximum gradient occurred, column 3 gives the value of the maximum gradient normalized by the nominal value of the element, and column 4 gives the projected percent change in the element required to produce a 10% reduction in the singular value. This 10% reduction in $\sigma_{\min}$ would correspond to a reduction in GM and PM of 0.83 dB and ± 3 deg, respectively. (The maximum singular value, on the other hand, is very large and would have to vary by more than 50% to become significant. Since this would clearly violate the linearity assumption implicit in this analysis, gradients with respect to the maximum singular values will not be presented here.) It is seen in column 3 that six elements, indicated by (   )**,

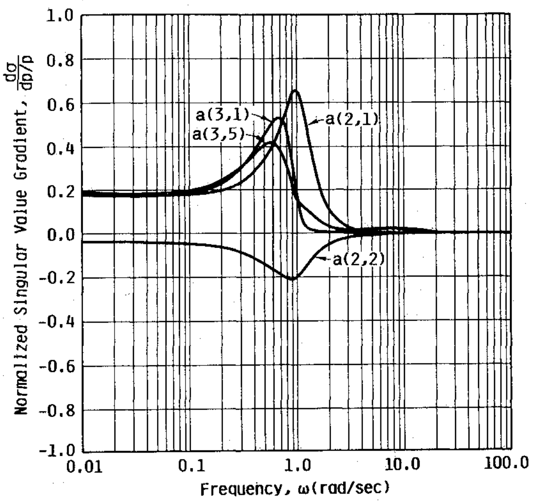

Fig. 7 Singular-value gradients with respect to elements $a(2,1)$, $a(3,1)$, and $a(3,5)$ of matrix A of the yaw/roll damper SAS.

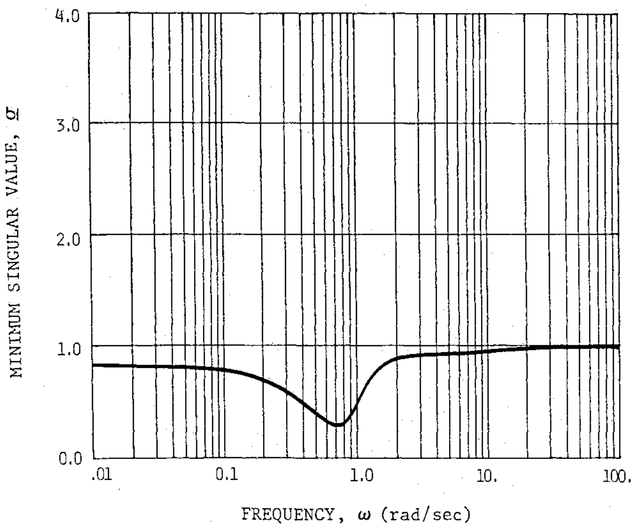

Fig. 8 Singular-value plot obtained by perturbing elements $a(2,7)$, $a(2,1)$, $a(2,2)$, $a(3,1)$, and $a(3,5)$ by 15% in their worst direction.

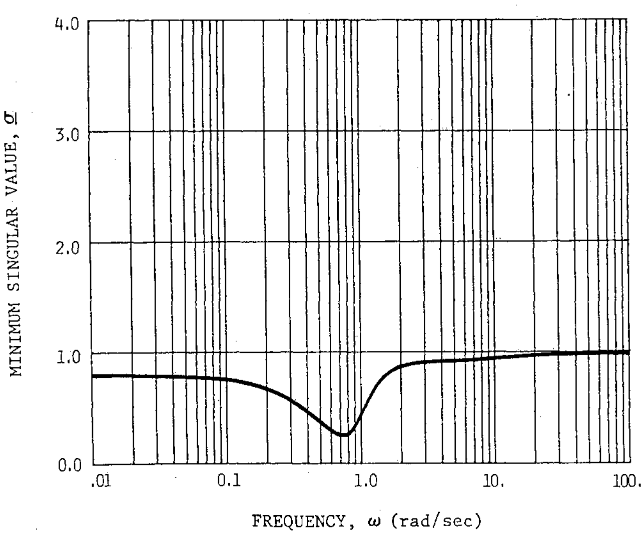

Fig. 9 Singular-value plot obtained by perturbing the 14 elements of column 1 in Table 3 by 15% in their worst direction.

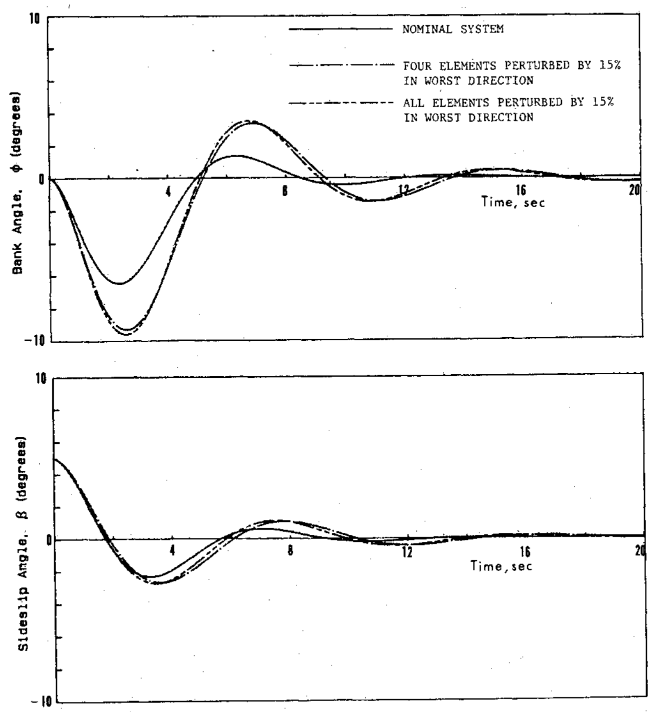

Fig. 10 Response of nominal system, five-element perturbed system, and total perturbed system for a β-5 deg initial condition.

Table 3 Summary of the sensitivity analysis information used in the selection of the important elements for the yaw/roll damper SAS

Element	(1) Element of A	(2) Frequency, rad/s	(3) $\partial\sigma/(\partial p/p)\mid_{max}$	(4) Approximated $\Delta p/p\mid_{10\%\sigma}$, %
1	$a(1,1)$	0.86	−0.091	−57
2	$a(1,3)$	1.12	(0.41)**	17
3	$a(1,4)$	0.95	−0.17	−34
4	$a(1,5)$	0.72	−0.014	−370
5	$a(2,1)$	0.95	(0.67)**	(9)**
6	$a(2,2)$	0.89	(−0.21)**	(−25)**
7	$a(2,3)$	0.73	0.021	240
8	$a(2,5)$	0.65	0.070	75
9	$a(2,7)$	0.86	(0.21)**	(25)**
10	$a(3,1)$	0.67	(0.54)**	(10)**
11	$a(3,2)$	0.56	−0.030	−190
12	$a(3,3)$	1.00	−0.095	−65
13	$a(3,5)$	0.56	(0.42)**	(13)**
14	$a(3,7)$	0.49	0.050	120

have large gradients. The selection of the cutoff value in judging if a gradient is large will require judgment by the designer. It is also important to note that these large gradients occur in the important frequency range ($0.1 \leq \omega \leq 3$ rad/s) and thus have the potential for altering the minimum singular value. Again, care must be taken in interpreting the results of the analysis. Consider element $a(1,3)$. Its singular-value gradient is one of the larger values, 0.41, and it is predicted that only a 17% change would be required to produce a 10% change in the minimum singular value. A closer examination of this element reveals, however, that it has the form

$$a(1,3) = (Y_r/U_1) - 1 \tag{13}$$

where Y_r is the dimensional stability derivative that relates the side force to yaw rate and U_1 the aircraft trim speed. Typically, Y_r/U_1 is small compared to -1; $Y_r/U_1 = 0.005$ for the present example. The -1 is a kinematic-type relation that would not be variable; thus, it is not realistic that the total $a(1,3)$ element would vary by 17%. Therefore, the $a(1,3)$, element is not highlighted in column 4. The sensitivity analysis has thus identified as key elements of the model $a(2,1)$, $a(2,2)$, $a(2,7)$, $a(2,1)$, and $a(3,5)$. These elements correspond to the parameters L_β, L_p, $L_{\delta A}$, N_β, and $N_{\delta R}$.That this result agrees with what one would expect attests to the strength of the technique: without any prior experience, one can accurately identify the most important parameters. This ability is very important when studying complex multiloop problems for which the answers to sensitivity questions are not so obvious. Other advantages of this technique over other methods are discussed in Ref. 10.

As a final check, the five elements selected in column four of Table 3 were allowed to change by 15% in their worst direction (so as to decrease $\underline{\sigma}$), and the singular-value plot of the perturbed closed-loop system was calculated. This plot is shown in Fig. 8. The minimum singular value is now given by $\underline{\sigma}=0.256$, which yields $a-1.98 \leq \text{GM} \leq 2.57$ dB and PM $= \pm 14.7$ deg. Comparing the perturbed system sigma plot ($\underline{\sigma}=0.256$, with the nominal system sigma plot of Fig. 5 ($\underline{\sigma}=0.502$), a reduction of 49% in the minimum singular value is observed. This reduction translates into the reduced stability margins. If now all 14 elements of A are allowed to change by the same 15% in their worst direction, the sigma plot of Fig. 9 is obtained, with a minimum singular value of $\underline{\sigma}=0.2292$, which yields $a-1.79 \leq \text{GM} \leq 2.26$ dB and a PM $= \pm 13.16$ deg. The minimum singular value has been reduced by 54% from the nominal system singular value, with 49% of the reduction due to the five elements selected as the most important and only 5% due to the remaining elements. As another way of illustrating that the key elements had been identified, Fig. 10 shows the response for an initial value of $\beta=5$ deg of the nominal system, the five-element perturbed system, and the system with all elements perturbed. Again, it can be seen that the major effects are due to the variation in the five elements identified in the analysis.

Conclusions

The sensitivity analysis for multivariable control designs that combines singular-value analysis and singular-value gradient analysis provides the control designer with a tool that can be used to identify when a vehicle model is adequate or, if not, what model refinements are required. One advantage of this formulation is that it presents the results in terms of gain and phase margins that are familiar to the designer. The sensitivity analysis does, however, require a thorough understanding of the model and controller characteristics. The ability to identify key model parameters should assist in the development of efficient test programs.

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